

Spin pumping by a field-driven domain wall

R. A. Duine

Institute for Theoretical Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands

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We present the theory of spin pumping by a field-driven domain wall for the situation that spin is not fully conserved. We calculate the pumped current in a metallic ferromagnet to first order in the time derivative of the magnetization direction. Irrespective of the microscopic details, the result can be expressed in terms of the conductivities of the majority and minority electrons and the dissipative spin transfer torque parameter β . The general expression is evaluated for the specific case of a field-driven domain wall and for that case depends strongly on the ratio of β and the Gilbert damping constant. These results may provide an experimental method to determine this ratio, which plays a crucial role for current-driven domain-wall motion.

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I. INTRODUCTION

Adiabatic quantum pumping of electrons in quantum dots^{1,2} has recently been demonstrated experimentally for both charge³ and spin.⁴ Currently, the activity in this field is mostly concentrated on the effects of interactions,⁵ dissipation,⁶ and nonadiabaticity.⁷ Complementary to these developments, the emission of spin current by a precessing ferromagnet—called spin pumping—has been studied theoretically and experimentally in single-domain magnetic nanostructures.^{8–10} One of the differences between spin pumping in single-domain ferromagnets and quantum pumping in quantum dots is that in the latter, the Hamiltonian of the electronic quasiparticles is manipulated directly, usually by varying the gate voltage of the dot. In the case of ferromagnets, however, it is the order parameter—the magnetization direction—that is driven by an external (magnetic) field. The coupling between the order parameter and the current-carrying electrons in turn pumps the spin current.¹¹ The opposite effect, i.e., the manipulation of magnetization with spin current, is called spin transfer.^{12–15}

Recently, the possibility of manipulating with current the position of a magnetic domain wall via spin transfer torques has attracted a great deal of theoretical^{16–30} and experimental^{31–38} interest. Although the subject is still controversial,^{18,21} it is by now established that in the long-wavelength limit, the equation of motion for the magnetization direction $\mathbf{\Omega}$, which in the absence of current describes damped precession around the effective field $-\delta E_{\text{MM}}[\mathbf{\Omega}]/(\hbar\delta\mathbf{\Omega})$, is given by

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) \mathbf{\Omega} - \mathbf{\Omega} \times \left(- \frac{\delta E_{\text{MM}}[\mathbf{\Omega}]}{\hbar\delta\mathbf{\Omega}} \right) \\ = - \mathbf{\Omega} \times \left(\alpha_G \frac{\partial}{\partial t} + \beta \mathbf{v}_s \cdot \nabla \right) \mathbf{\Omega} \end{aligned} \quad (1)$$

and contains, to lowest order in spatial derivatives of the magnetization direction, two contributions due the presence of electric current.

The first is the reactive spin transfer torque,^{16,17} which corresponds to the term proportional to $\nabla\mathbf{\Omega}$ on the left-hand side of the above equation. It is characterized by the velocity

\mathbf{v}_s that is linear in the current and related to the external electric field \mathbf{E} by

$$\mathbf{v}_s = \frac{(\sigma_{\downarrow} - \sigma_{\uparrow})\mathbf{E}}{|e|\rho_s}, \quad (2)$$

where σ_{\uparrow} and σ_{\downarrow} denote the conductivities of the majority and minority electrons, respectively, and ρ_s is their density difference. (The elementary charge is denoted by $|e|$.) The second term in Eq. (1) due to the current is the dissipative spin transfer torque³⁹ that is proportional to β .^{19–21} Both this parameter, and the Gilbert damping parameter α_G , have their microscopic origin in processes in the Hamiltonian that break conservation of spin, such as spin-orbit interactions.

It turns out that the phenomenology of current-driven domain-wall motion depends crucially on the value of the ratio β/α_G . For example, for $\beta=0$, the domain wall is intrinsically pinned,¹⁸ which means that there is a critical current even in the absence of inhomogeneities. For $\beta/\alpha_G=1$, on the other hand, the domain wall moves with velocity \mathbf{v}_s . Although theoretical studies indicate that generically $\beta \neq \alpha_G$,^{26–28,30} it is not well understood what the relative importance of spin-dependent disorder and spin-orbit effects in the bandstructure is, and a precise theoretical prediction of β/α_G for a specific material has not been attempted yet. Moreover, the determination of the ratio β/α_G from experiments on the current-driven domain-wall motion has turned out to be hard because of the extrinsic pinning of the domain and nonzero-temperature^{29,38} effects.

In this paper, we present the theory of the current pumped by a field-driven domain wall for the situation that spin is not conserved. In particular, we show that a field-driven domain wall in a metallic ferromagnet generates a charge current that depends strongly on the ratio β/α_G . This charge current arises from the fact that a time-dependent magnetization generates a spin current, similar to the spin-pumping mechanism proposed by Tserkovnyak *et al.*⁸ for nanostructures containing ferromagnetic elements. Since the symmetry between majority and minority electrons is by definition broken in a ferromagnet, this spin current necessarily implies a charge current. In view of this, we prefer to use the term “spin pumping” also for the case that spin is not fully conserved,

and defining the spin current as a conserved current is no longer possible.

The generation of spin and charge currents by a moving domain wall via electromotive forces is discussed very recently by Barnes and Maekawa.⁴⁰ We also note here the work by Ohe *et al.*,⁴¹ who consider the case of the Rashba model, and the very recent work by Saslow,⁴² Yang *et al.*,⁴³ and Tserkovnyak and Mecklenburg.⁴⁴ In addition to these recent papers, we mention the much earlier work by Berger, which discusses the current induced by a domain wall in terms of an analog of the Josephson effect.⁴⁵

Barnes and Maekawa⁴⁰ consider the case that spin is fully conserved. In this situation, it is convenient to perform a time- and position-dependent rotation in spin space, such that the spin quantization axis is locally parallel to the magnetization direction. As a result of spin conservation, the Hamiltonian in this rotated frame contains now only time-independent scalar and exchange potential terms. The kinetic-energy term of the Hamiltonian, however, will acquire additional contributions that have the form of a covariant derivative. Perturbation theory in these terms then amounts to performing a gradient expansion in the magnetization direction.¹⁷ Hence, the fact that Barnes and Maekawa consider the case that spin is fully conserved is demonstrated mathematically by noting that in Eq. (5) of Ref. 40, there are no time-dependent potential-energy terms. Generalizing this approach to the case of spin-dependent disorder or spin-orbit coupling turns out to be difficult. Nevertheless, Kohno and Shibata were able to determine the Gilbert damping and dissipative spin transfer torques using the above-mentioned method.⁴⁶ Since Barnes and Maekawa⁴⁰ consider the situation that spin is fully conserved, they are effectively dealing with the case that $\alpha_G = \beta = 0$. This is because both the Gilbert damping parameter α_G and the dissipative spin transfer torque parameter β arise from processes in the microscopic Hamiltonian that do not conserve spin.^{26–28,30} Hence, for the case that $\alpha_G = \beta = 0$, our results agree with the results of Barnes and Maekawa.⁴⁰

The remainder of this paper is organized as follows. In Sec. II, we derive a general expression for the electric current induced by a time-dependent magnetization texture. This general expression is then evaluated in Sec. III for a simple model of field-driven domain-wall motion. We end in Sec. IV with a short discussion and present our conclusions and outlook.

II. ELECTRIC CURRENT

Quite generally, the expectation value of the charge current density, defined by $\mathbf{j} = -c\partial H/\partial \mathbf{A}$ with c the speed of light, H the Hamiltonian, and \mathbf{A} the electromagnetic vector potential, is given as a functional derivative of the effective action

$$\langle \mathbf{j}(\mathbf{x}, \tau) \rangle = c \frac{\delta S_{\text{eff}}}{\delta \mathbf{A}(\mathbf{x}, \tau)}, \quad (3)$$

with τ the imaginary-time variable that runs from 0 to $\hbar/(k_B T)$. (Planck's constant is denoted by \hbar and $k_B T$ is the thermal energy.) First, we assume that spin is conserved,

which means that the Hamiltonian is invariant under rotations in spin space. The part of the effective action for the magnetization direction that depends on the electromagnetic vector potential is then given by¹⁷

$$S_{\text{eff}} = \int d\tau \int d\mathbf{x} \langle j_{s,\alpha}^z(\mathbf{x}, \tau) \rangle \tilde{A}_{\alpha'}(\boldsymbol{\Omega}(\mathbf{x}, \tau)) \nabla_{\alpha} \Omega_{\beta}(\mathbf{x}, \tau), \quad (4)$$

where a summation over Cartesian indices $\alpha, \alpha', \alpha'' \in \{x, y, z\}$ is implied throughout this paper. In this expression,

$$\begin{aligned} j_{s,\alpha'}^{\alpha}(\mathbf{x}, \tau) = & \frac{\hbar^2}{4mi} \{ \boldsymbol{\phi}^{\dagger}(\mathbf{x}, \tau) \tau^{\alpha} \nabla_{\alpha'} \boldsymbol{\phi}(\mathbf{x}, \tau) \\ & - [\nabla_{\alpha'} \boldsymbol{\phi}^{\dagger}(\mathbf{x}, \tau)] \tau^{\alpha} \boldsymbol{\phi}(\mathbf{x}, \tau) \} \\ & + \frac{|e|\hbar}{2mc} A_{\alpha'} \boldsymbol{\phi}^{\dagger}(\mathbf{x}, \tau) \tau^{\alpha} \boldsymbol{\phi}(\mathbf{x}, \tau) \end{aligned} \quad (5)$$

is the spin current, given here in terms of the Grassman coherent state spinor $\boldsymbol{\phi}^{\dagger} = (\phi_{\uparrow}^*, \phi_{\downarrow}^*)$. Furthermore, τ^{α} are the Pauli matrices and m is the electron mass. (Note that since we are, for the moment, considering the situation that spin is conserved, there are no problems regarding the definition of the spin current.) The expectation value $\langle \dots \rangle$ is taken with respect to the current-carrying collinear state of the ferromagnet. Finally, $\tilde{A}_{\alpha}(\boldsymbol{\Omega})$ is the vector potential of a magnetic monopole in spin space [not to be confused with the electromagnetic vector potential $\mathbf{A}(\mathbf{x}, \tau)$] that obeys $\epsilon_{\alpha\alpha'\alpha''} \partial \tilde{A}_{\alpha'} / \partial \Omega_{\alpha''} = \Omega_{\alpha}$ and is well known from the path-integral formulation for spin systems.⁴⁷ Equation (4) is most easily understood as arising from the Berry phase picked up by the spin of the electrons as they drift adiabatically through a noncollinear magnetization texture.^{16,17} Variation of this term with respect to the magnetization direction gives the reactive spin transfer torque in Eq. (1).

The expectation value of the spin current is given by

$$\langle j_{s,\alpha}^z(\mathbf{x}, \tau) \rangle = \int d\tau' \int d\mathbf{x}' \Pi_{\alpha,\alpha'}^z(\mathbf{x} - \mathbf{x}'; \tau - \tau') \frac{A_{\alpha'}(\mathbf{x}', \tau')}{\hbar c}. \quad (6)$$

The zero-momentum low-frequency part of the response function $\Pi_{\alpha,\alpha'}^z(\mathbf{x} - \mathbf{x}'; \tau - \tau') \equiv \langle j_{s,\alpha}^z(\mathbf{x}, \tau) j_{\alpha'}(\mathbf{x}', \tau') \rangle_0$, with $\langle \dots \rangle_0$ the equilibrium expectation value, is determined by noting that for the vector potential $\mathbf{A}(\mathbf{x}, \tau) = -c\mathbf{E}e^{-i\omega\tau}/\omega$, the above equation [Eq. (6)] should in the zero-frequency limit reduce to Ohm's law $\langle \mathbf{j}_s \rangle_0 = -\hbar(\sigma_{\uparrow} - \sigma_{\downarrow})\mathbf{E}/(2|e|)$. Using this result together with Eqs. (3)–(6), we find, after a Wick rotation $\tau \rightarrow it$ to real time, that

$$\langle j_{\alpha} \rangle = -\frac{\hbar}{2|e|V} (\sigma_{\uparrow} - \sigma_{\downarrow}) \frac{\partial}{\partial t} \int d\mathbf{x} \tilde{A}_{\alpha'}[\boldsymbol{\Omega}(\mathbf{x}, t)] \nabla_{\alpha} \Omega_{\alpha'}(\mathbf{x}, t), \quad (7)$$

with V the volume of the system. We note that the time derivative of the Berry phase term is also encountered by Barnes and Maekawa in discussing the electromotive force

in a ferromagnet.⁴⁰ Such Berry phase terms are known to occur in adiabatic quantum pumping.⁴⁸

We now generalize this result to the situation where spin is no longer conserved, for example, due to spin-orbit interactions or spin-dependent impurity scattering. Linearizing around the collinear state by means of $\mathbf{\Omega} = (\delta\Omega_x, \delta\Omega_y, 1 - \delta\Omega_x^2/2 - \delta\Omega_y^2/2)$, we find that the part of the effective action that contains the electromagnetic vector potential reads³⁰

$$S^{\text{eff}} = \int d\tau \int d\mathbf{x} \int d\tau' \int d\mathbf{x}' \int d\tau'' \int d\mathbf{x}'' \times [\delta\Omega_a(\mathbf{x}, \tau) \mathbf{K}_{ab}(\mathbf{x}, \mathbf{x}', \mathbf{x}'', \tau, \tau', \tau'') \cdot \mathbf{A}(\mathbf{x}'', \tau'') \delta\Omega_b(\mathbf{x}', \tau')], \quad (8)$$

where a summation over transverse indices $a, b \in \{x, y\}$ is implied. The spin-wave photon interaction vertex

$$\mathbf{K}_{ab}(\mathbf{x}, \mathbf{x}', \mathbf{x}'', \tau, \tau', \tau'') = \frac{\Delta^2}{8\hbar c} \langle \phi^\dagger(\mathbf{x}, \tau) \tau^a \phi(\mathbf{x}, \tau) \phi^\dagger(\mathbf{x}', \tau') \tau^b \phi(\mathbf{x}', \tau') \mathbf{j}(\mathbf{x}'', \tau'') \rangle_0, \quad (9)$$

given in terms of the exchange splitting Δ , is also encountered in a microscopic treatment of spin transfer torques.³⁰ The reactive part of this interaction vertex determines the reactive spin transfer torque and, via Eqs. (3) and (8), reproduces Eq. (7). The zero-frequency long-wavelength limit of the dissipative part of the spin-wave photon interaction vertex determines the dissipative spin transfer torque. (Note that in this approach, the definition of the spin current does not enter in determining the spin transfer torques.) Although Eq. (9) may be evaluated for a given microscopic model within some approximation scheme,³⁰ we need here only that variation of the action in Eq. (8) reproduces both the reactive and dissipative spin torques in Eq. (1). The final result for the electric current density is then given by

$$\langle j_a \rangle = - \frac{\hbar}{2|e|V} (\sigma_\uparrow - \sigma_\downarrow) \left\{ \beta \int d\mathbf{x} \frac{\partial \mathbf{\Omega}(\mathbf{x}, t)}{\partial t} \cdot \nabla_a \mathbf{\Omega}(\mathbf{x}, t) + \frac{\partial}{\partial t} \int d\mathbf{x} \tilde{A}_{a'}[\mathbf{\Omega}(\mathbf{x}, t)] \nabla_a \Omega_{a'}(\mathbf{x}, t) \right\}. \quad (10)$$

The above equation is essentially the result of a linear-response calculation in $\partial \mathbf{\Omega} / \partial t$ and is the central result of this paper. We emphasize that the way in which the transport coefficients σ_\uparrow and σ_\downarrow and the β parameter enter does not rely on the specific details of the underlying microscopic model. Note that the above result reduces to that of Barnes and Maekawa [Eq. (9) of Ref. 40] if we take $\beta=0$.

III. FIELD-DRIVEN DOMAIN-WALL MOTION

To bring out the qualitative physics, we evaluate the result in Eq. (10) using a simple model for field-driven domain-wall motion in a magnetic wire of length L . In polar coordinates θ and ϕ , defined by $\mathbf{\Omega} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, we choose the micromagnetic energy functional

$$E_{\text{MM}}[\theta, \phi] = \rho_s \int d\mathbf{x} \left\{ \frac{J}{2} [(\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2] + \frac{K_\perp}{2} \sin^2 \theta \sin^2 \phi - \frac{K_z}{2} \cos^2 \theta + gB \cos \theta \right\}, \quad (11)$$

where J is the spin stiffness and K_\perp and K_z are anisotropy constants larger than zero. The external field in the negative z direction leads to an energy splitting $2gB > 0$. We solve the equation of motion in Eq. (1) within the variational ansatz^{18,49}

$$\theta(\mathbf{x}, t) = \theta_0(\mathbf{x}, t) \equiv 2 \tan^{-1} [e^{-r_{\text{dw}}(t)-x/\lambda}], \quad (12)$$

together with $\phi(\mathbf{x}, t) = \phi_0(t)$, that describes a rigid domain wall with width $\lambda = \sqrt{J/K_z}$ at position $r_{\text{dw}}(t)$. The chirality of the domain wall is determined by the angle $\phi_0(t)$ and the magnetization direction is assumed to depend only on x , which is taken in the long direction of the wire.

The equations of motion for the variational parameters are given by^{18,29,49}

$$\dot{\phi}_0(t) + \alpha_G \left(\frac{\dot{r}_{\text{dw}}(t)}{\lambda} \right) = \frac{gB}{\hbar},$$

$$\left(\frac{\dot{r}_{\text{dw}}(t)}{\lambda} \right) - \alpha_G \dot{\phi}_0(t) = \frac{K_\perp}{2\hbar} \sin 2\phi_0(t). \quad (13)$$

Note that the velocity \mathbf{v}_s is absent from these equations since we consider the generation of electric current by a field-driven domain wall. The above equations provide a description of the field-driven domain wall and, in particular, of Walker breakdown.⁴⁹ That is, for an external field smaller than the Walker breakdown field $B_W \equiv \alpha_G K_\perp / (2g)$, the domain wall moves with a constant velocity. For fields $B > B_W$, the domain wall undergoes oscillatory motion, which initially makes the average velocity smaller.

Solving the equations of motion results in

$$\overline{\dot{\phi}_0} = \frac{1}{(1 + \alpha_G^2)} \text{Re} \left[\sqrt{\left(\frac{gB}{\hbar} \right)^2 - \left(\frac{\alpha_G K_\perp}{2\hbar} \right)^2} \right],$$

$$\frac{\overline{\dot{r}_{\text{dw}}}}{\lambda} = \frac{gB}{\alpha_G \hbar} - \frac{\overline{\dot{\phi}_0}}{\alpha_G}, \quad (14)$$

where the $\overline{\dots}$ indicates taking the time-averaged value. Inserting the variational ansatz into Eq. (10) leads in first instance to

$$\langle j_x \rangle = - \frac{\hbar}{|e|L} (\sigma_\uparrow - \sigma_\downarrow) \left[\frac{\beta \overline{\dot{r}_{\text{dw}}(t)}}{\lambda} + \overline{\dot{\phi}_0(t)} \right], \quad (15)$$

which, using Eq. (14), becomes

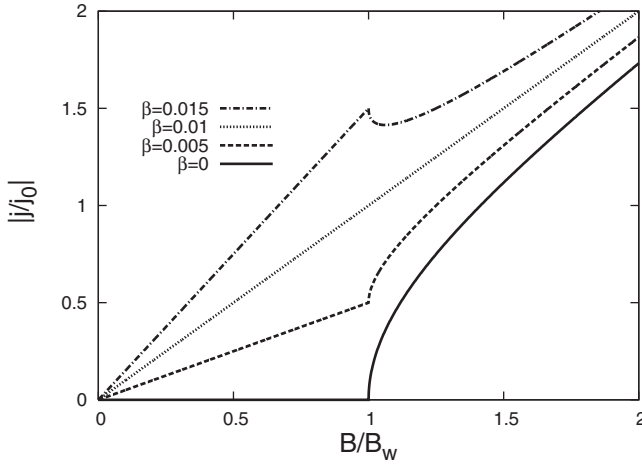


FIG. 1. Current generated by a field-driven domain wall in units of $j_0 = 2L/[e(\sigma_\uparrow - \sigma_\downarrow)\alpha_G K_\perp]$ for $\alpha_G = 0.01$ and various values of β . The result is plotted as a function of magnetic field in units of the Walker breakdown field $B_W \equiv \alpha_G K_\perp / (2g)$.

$$\begin{aligned} \overline{\langle j_x \rangle} = & -\frac{\hbar}{|e|L}(\sigma_\uparrow - \sigma_\downarrow) \left\{ \frac{\beta g B}{\alpha_G \hbar} + \left(\frac{1 - \beta}{1 + \alpha_G^2} \right) \right. \\ & \left. \times \text{Re} \left[\sqrt{\left(\frac{gB}{\hbar} \right)^2 - \left(\frac{\alpha_G K_\perp}{2\hbar} \right)^2} \right] \right\}. \end{aligned} \quad (16)$$

As shown in Fig. 1, this result depends strongly on the ratio β/α_G . In particular, for $\beta > \alpha_G$, a local maximum appears in the current as a function of magnetic field. Since α_G is determined independently from ferromagnetic resonance experiments, measurement of the slope of the current for small magnetic fields enables experimental determination of β . We note that within the present approximation, the current does not depend on the domain-wall width λ . Furthermore, in the limit of zero Gilbert damping and β , the dissipationless limit, we have that the current density is equal to $\overline{\langle j_x \rangle} = (\sigma_\downarrow - \sigma_\uparrow)gB/(|e|L)$. This is the result of Barnes and Maekawa⁴⁰ that corresponds to the situation that $\alpha_G = \beta = 0$, as discussed in the Introduction. We point out that, within our approximation for the description of domain-wall motion, setting $\beta = \alpha_G$ in Eq. (16) gives the same result as using Eqs. (13) and (15) with $\alpha_G = \beta = 0$. That the situation discussed by Barnes and Maekawa⁴⁰ is indeed that of $\alpha_G = \beta = 0$ is seen by comparing their result [Eqs. (8) and (9) of Ref. 40 and the paragraph following Eq. (9)] with our results in Eqs. (10) and (13).

IV. DISCUSSION AND CONCLUSIONS

Our result in Eq. (16) is a simple expression for the pumped current as a function of magnetic field for a field-

driven domain wall. A possible disadvantage in using Eq. (16), however, is that in deriving this result, we assumed a specific model to describe the motion of the domain wall. This model does, in first instance, not include extrinsic pinning and nonzero temperature. Both extrinsic pinning¹⁸ and nonzero temperature²⁹ can be included in the rigid-domain-wall description. However, it is in some circumstances perhaps more convenient to directly use the result in Eq. (15) together with the experimental determination of $\dot{r}_{\text{dw}}(t)$. Since the only way in which the parameter β enters this equation is as a prefactor of $\dot{r}_{\text{dw}}(t)$, this should be sufficient to determine its value from the experiment. We note, however, that the precision with which the ratio β/α_G can be determined depends on how accurately the magnetization dynamics and, in particular, the motion of the domain wall are imaged experimentally. With respect to this, we note that the various curves in Fig. 1 are qualitatively different for different values of β/α_G . In particular, the results for $\beta/\alpha_G > 1$ and $\beta/\alpha_G < 1$ differ substantially and could most likely be experimentally distinguished. In view of this discussion, future research will in part be directed toward evaluating Eq. (10) for more complicated models of field-driven domain-wall motion, which will benefit the experimental determination of β/α_G .

A typical current density is estimated as follows. For the experiments of Beach *et al.*,⁵⁰ we have that $L \sim 20 \mu\text{m}$ and $\lambda \sim 20 \text{nm}$. The domain velocities measured in this experiment are $\dot{r}_{\text{dw}} \sim 40\text{--}100 \text{m/s}$. Taking as a typical conductivity $\sigma_\uparrow \sim 10^6 \Omega^{-1} \text{m}^{-1}$, we find, using Eq. (15) with $\beta \sim 0.01$, typical electric current densities of the order of $\langle j_x \rangle \sim 10^3\text{--}10^4 \text{A m}^{-2}$. This result depends somewhat on the polarization of the electric current in the ferromagnetic metal, which we have taken equal to 50%–100% in this rough estimate. Although much smaller than typical current densities required to move the domain wall via spin transfer torques, electrical current densities of this order appear to be detectable experimentally.

In conclusion, we have presented a theory of spin pumping without spin conservation and, in particular, proposed a way to gain experimental access to the parameter β/α_G that is of great importance for the physics of current-driven domain-wall motion. We note that the mechanism for current generation discussed in this paper is quite distinct from the generation of eddy currents by a moving magnetic domain.⁵¹ In addition to improving upon the model used for describing domain-wall motion, we intend to investigate in future work whether the damping terms in Eq. (1) or possible higher-order terms in frequency and momentum⁵² have a natural interpretation in terms of spin pumping, similar to the spin-pumping-enhanced Gilbert damping in single-domain ferromagnets.⁸

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